

Threshold-Resummed Cross Section for the Drell-Yan Process in Pion-Nucleon Collisions at COMPASS

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We present a study of the Drell-Yan process in pion-proton collisions including next-to-leading-logarithmic threshold-resummed contributions. We analyze rapidity-integrated as well as rapidity-differential cross sections in the kinematic regime relevant for the COMPASS fixed target experiment. We find that resummation leads to a significant enhancement of the cross section compared to fixed-order calculations in this regime. Particularly large corrections arise at large forward and backward rapidities of the lepton pair. We also study the scale dependence of the cross section and find it to be substantially reduced by threshold resummation.

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I. INTRODUCTION

Drell-Yan lepton pair production by incident charged pions is still the only available hadronic process that allows for the extraction of the internal “partonic” structure of the pion in the valence region. Unfortunately, experimental data for this process are rather scarce [1, 2]. Several analyses of the available data have been performed using leading order (LO) or next-to-leading order (NLO) partonic cross sections, with the goal of extracting the parton distribution functions of the pion [3–5]. The resulting LO or NLO valence distributions showed a linear ($\sim (1-x)^1$) or slightly faster fall-off at high x , at odds with theoretical predictions based on perturbative QCD [6], and calculations using Dyson-Schwinger equations [7], which prefer a much softer behavior $\sim (1-x)^2$ at high x . For a review, see [8].

In a recent analysis [9] we showed that a valence distribution with a fall-off $\sim (1-x)^{2\pm 0.1}$ is in fact well consistent with the Drell-Yan data, *if* large logarithmic contributions arising near the threshold for the partonic reaction are taken into account to all orders in perturbation theory. These so-called “threshold logarithms” strongly enhance the partonic cross section near threshold, so that a softer valence distribution is sufficient to describe the data. We also found in Ref. [9] that the available pionic Drell-Yan data are not able to completely determine the valence parton distribution of the pion. The data are almost equally well described by a valence distribution that carries 60 %, 65 % or 70 % of the pion’s momentum at a low input scale $Q_0 = 0.63$ GeV, as long as it behaves approximately as $(1-x)^2$ at high x . The quality of three corresponding fits that we performed in [9] only differed by about one unit in χ^2 . This uncertainty in the valence momentum is also manifest in the earlier NLO analyses. At $Q = 2$ GeV the valence parton distribution of SMRS [3] carries 46 % of the pion’s momentum, whereas the valence distribution of GRS [4] carries only 40 %, although both distributions describe the same data sets

equally well. The origins of this ambiguity are the large overall systematic uncertainties of the Drell-Yan data. These uncertainties can be best discussed by introducing an additional K -factor that multiplies the theoretical cross section. The numerical value of this K -factor is strongly correlated with the first moment of the chosen valence distribution (see Table I of Ref. [9]). Hence, pion Drell-Yan data with a well understood normalization are urgently needed to really pin down the pion’s valence distribution.

The upcoming fixed-target πN Drell-Yan experiment [10] at COMPASS is hoped to resolve this issue. In this paper, we present detailed predictions for the rapidity-integrated as well as the rapidity-differential cross section for the kinematics relevant at COMPASS. In the light of our study [9], it is expected that threshold logarithms will also play a significant role for the Drell-Yan cross section at COMPASS and lead to large corrections. We will therefore base our predictions on the threshold resummation technique. We note that there have been numerous earlier phenomenological applications of threshold resummation in the Drell-Yan process [11–17], both for fixed-target and for collider energies. The specific application to πN scattering, and the resulting phenomenological studies for COMPASS, are the new elements of this paper.

The remainder of this paper is organized as follows. In Sec. II the basic framework for the calculation of the rapidity-differential Drell-Yan cross section is presented. We discuss fixed-order corrections as well as the full next-to-leading logarithmic resummation of threshold logarithms. This includes the discussion of Mellin and Fourier moments of the cross section, which are useful tools for threshold resummation. In Sec. III we present phenomenological results for the kinematic regime of the Drell-Yan experiment at COMPASS. We show both rapidity-integrated and rapidity-differential cross sections. We finally draw our conclusions in Sec. IV.

II. THEORETICAL FRAMEWORK

We follow in this section the framework laid out in Ref. [12], where threshold resummation effects in W^\pm boson production at hadron colliders were studied. We consider the inclusive cross section for the production of a $\mu^+\mu^-$ pair of invariant mass Q and rapidity η in the process

$$\pi^-(P_1)p(P_2) \rightarrow \mu^+\mu^-X, \quad (1)$$

where P_1 and P_2 are the four-momenta of the initial-state particles. According to the relevant factorization theorem, at high Q the rapidity-differential cross section may be written as

$$\begin{aligned} \frac{d\sigma}{dQ^2 d\eta} &= \sigma_0 \sum_{a,b} \int_{x_1^0}^1 \frac{dx_1}{x_1} \int_{x_2^0}^1 \frac{dx_2}{x_2} f_a^\pi(x_1, \mu^2) f_b^p(x_2, \mu^2) \\ &\times e_{ab} \omega_{ab}(x_1, x_1^0, x_2, x_2^0, Q/\mu), \end{aligned} \quad (2)$$

where $\sigma_0 = 4\pi\alpha^2/9Q^2S$ with $S = (P_1 + P_2)^2$ the hadronic center-of-mass energy squared. The coupling e_{ab} equals e_q^2 for the $q\bar{q}$ and qg , $\bar{q}g$ scattering processes which we are interested in, where e_q denotes the quark's fractional electromagnetic charge. In terms of the rapidity η the lower bounds of the x_1 and x_2 integrals are

$$x_{1,2}^0 = \sqrt{\tau} e^{\pm\eta}, \quad (3)$$

with $\tau = Q^2/S$. The sum in Eq. (2) runs over all partonic channels, with f_a^π and f_b^p the corresponding parton distribution functions (PDFs) of the pion and the proton, and ω_{ab} the hard-scattering functions. The latter can be computed in perturbation theory as series in the strong coupling constant α_s . The parton distribution functions as well as the hard-scattering functions depend on the factorization and renormalization scales, which we choose to be equal and collectively denote as μ .

At leading order $\mathcal{O}(\alpha_s^0)$ only the quark-antiquark annihilation channel $q\bar{q} \rightarrow \gamma^* \rightarrow \mu^+\mu^-$ contributes, for which one has in our normalization

$$\omega_{q\bar{q}}^{(0)} = x_1 x_2 \delta(x_1 - x_1^0) \delta(x_2 - x_2^0). \quad (4)$$

At next-to-leading order, apart from the $\mathcal{O}(\alpha_s)$ corrections to the $q\bar{q}$ process, also additional processes contribute to the cross section, namely $qg \rightarrow \gamma^*q$ and $\bar{q}g \rightarrow \gamma^*\bar{q}$. The NLO partonic cross sections in the $\overline{\text{MS}}$ scheme, which is the scheme we adopt throughout this work, can be obtained from [18]. They are also collected in the Appendix of Ref. [3].¹

As mentioned in the Introduction, the Drell-Yan cross section receives large logarithmic corrections near the threshold for the partonic reaction [20]. This threshold is defined by $z = Q^2/x_1 x_2 S = 1$, where x_1 and x_2 are the momentum fractions of the partons participating in the hard-scattering reaction. As z increases towards unity, most of the initial partonic energy is used to produce the virtual photon. Therefore, little phase space remains for real-gluon radiation, while virtual-gluon diagrams may still contribute fully. The infrared cancellations between the virtual and the “inhibited” real-emission diagrams then leave behind large logarithmic corrections to $\omega_{q\bar{q}}$. The leading terms among the resulting threshold logarithms are of the form $\alpha_s^k \ln^{2k-1}(1-z)/(1-z)$ at the k th order of perturbation theory. Subleading terms are down by one or more powers of the logarithm. The threshold logarithms become particularly important when $\tau = Q^2/S$, the hadronic analog of z , is large, which is generally the case in the fixed-target regime. The fact that the parton distribution functions are steeply falling functions of x_1 or x_2 emphasizes the threshold region in the cross section even for values of τ substantially smaller than one. In this kinematic regime the logarithms $\ln^{2k-1}(1-z)/(1-z)$ compensate the smallness of α_s^k , and it becomes necessary to resum the large corrections to all orders in the strong coupling. Such “threshold resummation” has originally been derived for the Drell-Yan process and deep inelastic scattering a long time ago [20]. The techniques developed in these seminal papers have been extended and successfully applied to the resummation of large logarithmic contributions in numerous other hard QCD processes.

Threshold resummation may be achieved in Mellin moment space, where phase space integrals for multiple-soft-gluon emission decouple. For the rapidity dependent cross section, it is convenient to also apply a Fourier transform in η [12, 21] (alternatively, one can also use a double Mellin transform [22]). Under combined Fourier and Mellin transforms of the cross section,

$$\sigma(N, M) \equiv \int_0^1 d\tau \tau^{N-1} \int_{-\ln \frac{1}{\sqrt{\tau}}}^{\ln \frac{1}{\sqrt{\tau}}} d\eta e^{iM\eta} \frac{d\sigma}{dQ^2 d\eta}, \quad (5)$$

the convolution integrals in (2) decouple into ordinary products [12, 21]. Defining the moments of the PDFs,

$$f^N(\mu^2) \equiv \int_0^1 dx x^{N-1} f(x, \mu^2), \quad (6)$$

and introducing the corresponding double transform of the partonic hard-scattering cross sections,

$$\tilde{\omega}_{ab}(N, M) \equiv \int_0^1 dz z^{N-1} \int_{-\ln \frac{1}{\sqrt{z}}}^{\ln \frac{1}{\sqrt{z}}} d\hat{\eta} e^{iM\hat{\eta}} \omega_{ab}, \quad (7)$$

where $\hat{\eta} = \eta - \frac{1}{2} \ln(x_1/x_2)$ is the partonic center-of-mass rapidity, one finds:

$$\sigma(N, M) = \sigma_0 \sum_{a,b} f_a^{\pi, N+i\frac{M}{2}} f_b^{p, N-i\frac{M}{2}} e_{ab} \tilde{\omega}_{ab}(N, M). \quad (8)$$

¹ We note that Refs. [3, 18] adopt a non-standard polarization average for incoming gluons in dimensional regularization. In order to correct for this, one simply needs to multiply the arguments of the logarithms in Eqs. (A8) and (A20) of [3] by e^{-1} [19].

The double transform thus factorizes the PDFs and the perturbatively calculable hard-scattering functions. The leading order contribution to the hard-scattering function $\tilde{\omega}_{ab}(N, M)$ is easily calculated by making use of the relations

$$\frac{x_1^0}{x_1} = \sqrt{z}e^{\hat{\eta}}, \quad \frac{x_2^0}{x_2} = \sqrt{z}e^{-\hat{\eta}}. \quad (9)$$

We obtain from Eq. (4) for the Fourier transform of $\omega_{q\bar{q}}^{(0)}$:

$$\begin{aligned} & \int_{-\ln(1/\sqrt{z})}^{\ln(1/\sqrt{z})} d\hat{\eta} e^{iM\hat{\eta}} x_1 x_2 \delta(x_1 - x_1^0) \delta(x_2 - x_2^0) \\ &= \int_{-\ln(1/\sqrt{z})}^{\ln(1/\sqrt{z})} d\hat{\eta} e^{iM\hat{\eta}} \delta(1 - \sqrt{z}e^{\hat{\eta}}) \delta(1 - \sqrt{z}e^{-\hat{\eta}}) \quad (10) \\ &= \cos(M \ln(1/\sqrt{z})) \delta(1 - z). \end{aligned}$$

Here we have appropriately averaged over the two possible solutions for the integral. The emerging factor $\delta(1-z)$ in Eq. (10) is just the LO hard-scattering contribution to the rapidity-integrated Drell-Yan cross section. Hence, the Fourier transform of the LO rapidity-differential partonic cross section is equal to the LO rapidity-integrated partonic cross section times $\cos(M \ln(1/\sqrt{z}))$ [12]. In the near-threshold limit $z \rightarrow 1$ this cosine factor becomes subleading:

$$\cos(M \ln(1/\sqrt{z})) = 1 - \frac{(1-z)^2 M^2}{8} + \mathcal{O}((1-z)^4 M^4). \quad (11)$$

As was discussed in Refs. [12, 13, 23], even at higher orders the dependence of the double moments $\tilde{\omega}_{ab=q\bar{q}}(N, M)$ on M becomes subleading near threshold, whereas the N -dependence is identical to that of the rapidity-integrated cross section. Therefore, the resummed expression for $\tilde{\omega}_{ab=q\bar{q}}(N, M)$ is equal to that for the total (rapidity-integrated) cross section. It was shown in Ref. [12] that keeping the cosine term in Eq. (10) in the resummed $\tilde{\omega}_{ab=q\bar{q}}$ slightly more faithfully reproduces the rapidity dependence at each order of perturbation theory. This can be easily achieved by writing the cosine as

$$\cos(M \ln(1/\sqrt{z})) = \frac{1}{2} \left(z^{iM/2} + z^{-iM/2} \right), \quad (12)$$

which, when combined with Eq. (7), leads to a sum of two terms with Mellin moments shifted to $N \pm iM/2$.

Threshold resummation for the Drell-Yan process results in the exponentiation of the soft-gluon corrections. To next-to-leading-logarithmic (NLL) order the resummed cross section is given in the $\overline{\text{MS}}$ scheme by

$$\begin{aligned} \ln \tilde{\omega}_{q\bar{q}} &= C_q \left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) + 2 \int_0^1 d\zeta \frac{\zeta^{N-1} - 1}{1 - \zeta} \\ &\times \int_{\mu^2}^{(1-\zeta)^2 Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} A_q(\alpha_s(k_{\perp})), \quad (13) \end{aligned}$$

where $A_q(\alpha_s)$ is a perturbative function, the $\mathcal{O}(\alpha_s^2)$ part of which is sufficient for resummation to NLL [20]:

$$A_q(\alpha_s) = \frac{\alpha_s}{\pi} A_q^{(1)} + \left(\frac{\alpha_s}{\pi} \right)^2 A_q^{(2)} + \dots, \quad (14)$$

with [24]

$$A_q^{(1)} = C_F, \quad A_q^{(2)} = \frac{1}{2} C_F \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \right]. \quad (15)$$

Here $C_F = 4/3$, $C_A = 3$. The first term in Eq. (13) does not originate from soft-gluon emission but instead mostly contains hard virtual corrections. It is also a perturbative series in α_s , and we only need its first-order term:

$$C_q = \frac{\alpha_s}{\pi} C_F \left(-4 + \frac{2\pi^2}{3} + \frac{3}{2} \ln \frac{Q^2}{\mu^2} \right) + \mathcal{O}(\alpha_s^2), \quad (16)$$

whose exponentiated form has been established in Ref. [25].

Since the perturbative running coupling $\alpha_s(k_{\perp})$ diverges at $k_{\perp} = \Lambda_{\text{QCD}}$, Eq. (13) as it stands is ill-defined. The perturbative expansion of the expression shows factorial divergence, which in QCD corresponds to a powerlike ambiguity of the series [26]. It turns out, however, that the factorial divergence appears only at nonleading powers of the momentum transfer. The large logarithms we are resumming arise in the region [20] $z \leq 1 - 1/\bar{N}$ in the integrand of the second term in Eq. (13). One therefore finds that to NLL they are contained in the simpler expression

$$\begin{aligned} & 2 \int_{Q^2/\bar{N}^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} A_q(\alpha_s(k_{\perp})) \ln \frac{\bar{N} k_{\perp}}{Q} \\ & + 2 \int_{Q^2}^{\mu^2} \frac{dk_{\perp}^2}{k_{\perp}^2} A_q(\alpha_s(k_{\perp})) \ln \bar{N} \quad (17) \end{aligned}$$

for the second term in Eq. (13), where $\bar{N} = N e^{\gamma_E}$ with the Euler constant γ_E . This form is used for “minimal” expansions [27] of the resummed exponent. From Eq. (17) one obtains for the resummed exponent to NLL accuracy [27, 28]:

$$\ln \tilde{\omega}_{q\bar{q}} = C_q + 2h^{(1)}(\lambda) \ln \bar{N} + 2h^{(2)} \left(\lambda, \frac{Q^2}{\mu^2} \right), \quad (18)$$

where

$$\lambda = b_0 \alpha_s(\mu^2) \ln \bar{N}. \quad (19)$$

The functions $h^{(1)}$, $h^{(2)}$ collect all leading-logarithmic and NLL terms in the exponent, which are of the form

$\alpha_s^k \ln^{k+1} \bar{N}$ and $\alpha_s^k \ln^k \bar{N}$, respectively. They read

$$\begin{aligned} h^{(1)}(\lambda) &= \frac{A_q^{(1)}}{2\pi b_0 \lambda} [2\lambda + (1-2\lambda) \ln(1-2\lambda)], \\ h^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}\right) &= -\frac{A_q^{(2)}}{2\pi^2 b_0^2} [2\lambda + \ln(1-2\lambda)] \\ &\quad + \frac{A_q^{(1)} b_1}{2\pi b_0^3} [2\lambda + \ln(1-2\lambda) \\ &\quad + \frac{1}{2} \ln^2(1-2\lambda)] \\ &\quad + \frac{A_q^{(1)}}{2\pi b_0} [2\lambda + \ln(1-2\lambda)] \ln \frac{Q^2}{\mu^2} \\ &\quad - \frac{A_q^{(1)} \alpha_s(\mu^2)}{\pi} \ln(\bar{N}) \ln \frac{Q^2}{\mu^2}, \end{aligned} \quad (20)$$

where

$$b_0 = \frac{1}{12\pi} (11C_A - 2N_f) \quad (21)$$

$$b_1 = \frac{1}{24\pi^2} (17C_A^2 - 5C_A N_f - 3C_F N_f). \quad (22)$$

The last term of the function $h^{(2)}$ depends on the factorization scale and compensates the evolution of the parton distribution functions. The scale dependence of the second-to-last term results from the running of the strong coupling constant. Since scale evolution exponentiates and is therefore taken into account to all orders, one expects [29, 30] a significant decrease in the scale dependence of the resummed cross section compared to a fixed order cross section.

As was shown in Refs. [31–33], it is possible to improve the above formula by taking into account certain subleading terms in the resummation. As in Ref. [12] we rewrite Eqs. (18)–(20) as

$$\begin{aligned} \ln \tilde{\omega}_{q\bar{q}} &= \frac{1}{\pi b_0} [2\lambda + \ln(1-2\lambda)] \left(\frac{A_q^{(1)}}{b_0 \alpha_s(\mu^2)} - \frac{A_q^{(2)}}{\pi b_0} + \frac{A_q^{(1)} b_1}{b_0^2} + A_q^{(1)} \ln \frac{Q^2}{\mu^2} \right) \\ &\quad + \frac{\alpha_s(\mu^2)}{\pi} C_F \left(-4 + \frac{2\pi^2}{3} \right) + \frac{A_q^{(1)} b_1}{2\pi b_0^3} \ln^2(1-2\lambda) + B_q^{(1)} \frac{\ln(1-2\lambda)}{\pi b_0} \\ &\quad + [-2A_q^{(1)} \ln \bar{N} - B_q^{(1)}] \left(\frac{\alpha(\mu^2)}{\pi} \ln \frac{Q^2}{\mu^2} + \frac{\ln(1-2\lambda)}{\pi b_0} \right), \end{aligned} \quad (23)$$

where $B_q^{(1)} = -3C_F/2$. The last term in Eq. (23) is the leading-logarithmic expansion of the integral

$$\int_{\mu^2}^{Q^2/\bar{N}^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{\alpha_s(k_{\perp}^2)}{\pi} [-2A_q^{(1)} \ln \bar{N} - B_q^{(1)}]. \quad (24)$$

The term in square brackets is the leading term in the large- N limit of the anomalous dimension of the one-loop diagonal ($q \rightarrow q$) splitting function P_{qq}^N , i.e. it governs the evolution of the parton distributions between scales μ and Q/\bar{N} . Replacing it by the full flavor nonsinglet LO splitting function [32],

$$[-2A_q^{(1)} \ln \bar{N} - B_q^{(1)}] \rightarrow C_F \left[\frac{3}{2} - 2S_1(N) + \frac{1}{N(N+1)} \right], \quad (25)$$

fully reproduces the diagonal part of the quark and anti-quark evolution. This replacement further reduces the scale dependence of the resummed cross section. We could also include a non-diagonal contribution from $g \rightarrow q$ splitting, corresponding to singlet mixing. However,

this contribution turns out to be numerically unimportant for the Drell-Yan process in the present kinematics.

As the exponentiation of soft-gluon corrections is achieved in Mellin moment and Fourier space, the hadronic cross section differential in Q^2 and η is obtained by taking the inverse Mellin and Fourier transforms of Eq. (8):

$$\frac{d\sigma}{dQ^2 d\eta} = \int_{-\infty}^{\infty} \frac{dM}{2\pi} e^{-iM\eta} \int_{C-i\infty}^{C+i\infty} \frac{dN}{2\pi i} \tau^{-N} \sigma(N, M). \quad (26)$$

When performing the inverse Mellin transform, the parameter C usually has to be chosen in such a way that all singularities of the integrand lie to the left of the integration contour. The resummed cross section, however, has a Landau singularity at $\lambda = 1/2$ or $\bar{N} = \exp(1/2\alpha_s b_0)$, as a result of the divergence of the running coupling α_s in Eq. (13) for $k_{\perp} \rightarrow \Lambda_{\text{QCD}}$. For the Mellin inversion, we adopt the *minimal prescription* developed in Ref. [27] to deal with the Landau pole. For this prescription the contour is chosen to lie to the *left* of the Landau sin-

gularity. Above and below the real axis, the contour is tilted into the half-plane with negative real part. This improves the convergence of the integration, since contributions with negative real part are exponentially suppressed by the factor τ^{-N} in Eq. (26). As mentioned earlier, the moment-space singularities of the parton distribution functions are shifted by $\pm iM/2$ from the real axis due to the Fourier transform. Ref. [21] provides details for how to prevent the tilted contour from passing through or below those singularities. We note that an alternative possibility for dealing with the Landau singularity is to perform the resummation directly in z -space [14].

We match the resummed cross section to the NLO one by subtracting the $O(\alpha_s)$ expansion of the resummed expression and adding the full NLO cross section [12]. This “matched” cross section consequently not only resums the large threshold logarithms to all orders, but also contains the full NLO results for the $q\bar{q}$ and qg channels. We will occasionally also consider a resummed cross section that has not been matched to the NLO one. We will refer to such a cross section as “unmatched”.

III. PHENOMENOLOGICAL RESULTS

We now present our numerical results for the Drell-Yan cross section at COMPASS. We will consider both the rapidity-integrated and the rapidity-differential hadronic cross section. Our main goal is to investigate the size of the threshold resummation effects. The π^- beam foreseen at COMPASS has an energy of 190 GeV. It is scattered off a proton target at rest, so that the resulting center-of-mass energy of the system is $\sqrt{S} \approx 19$ GeV.

For the pionic parton distribution functions we use the ones for the “preferred fit” of our previous study [9]. We remind the reader that these were extracted from a fit to the earlier πN Drell-Yan data [1, 2], using NLL threshold-resummed cross sections in the $\overline{\text{MS}}$ scheme. For the proton target we use the NLO ($\overline{\text{MS}}$ scheme) CTEQ6M [34] parton distributions. Unless stated otherwise, we choose the renormalization and factorization scales as $\mu = Q$.

We start by considering the differential cross section $d\sigma/dQ$, integrated over all rapidities, to show the relevance and the validity of the resummation effects over the whole range of the invariant mass Q . Here we ignore for simplicity charmonium and bottomonium resonances in the lepton pair spectrum, whose contributions are dominant for resonant invariant masses, and calculate only the smooth (continuum) part of the cross section. Figure 1 shows the cross section $Q^3 d\sigma/dQ$ at $\sqrt{S} = 19$ GeV at fixed order (LO and NLO), as well as for the NLL-resummed case. It can be seen that resummation leads to a significant enhancement of the cross section over LO, which increases strongly with invariant mass. This becomes even more apparent in Fig. 2, where we show the “K-factor”, defined as the ratio of the cross section to the

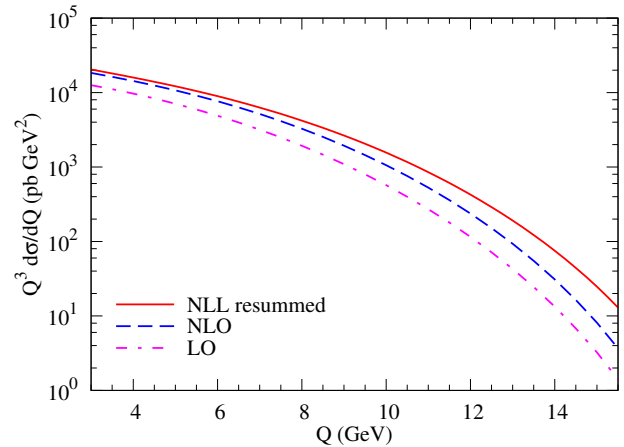


FIG. 1: Rapidity-integrated Drell-Yan cross section $Q^3 d\sigma/dQ$ for $\pi^- p$ scattering at $\sqrt{S} = 19$ GeV, at LO, NLO and NLL-resummed, as a function of the invariant mass Q of the lepton pair.

LO one:

$$K = \frac{d\sigma/dQ}{d\sigma^{\text{LO}}/dQ}. \quad (27)$$

The “K-factor” is plotted for the NLO and the NLL-resummed result. We also expand the *unmatched* resummed cross section in powers of α_s . The results for the first, second and third order expansion are also shown in Fig. 2. One can see that in the fixed-target regime higher orders (beyond NLO) still make large contributions to the cross section, especially at high invariant mass Q . This finding is in line with that in the earlier study [11] for $\bar{p}p$ -scattering. We also observe that the exact NLO cross section agrees extremely well with the first order expansion of the unmatched resummed result. This demonstrates that the logarithmic contributions from soft gluon radiation, which we resum to all orders, give by far the most important contribution to the cross section, not only very close to threshold as $\tau = Q^2/S \rightarrow 1$, but also for rather moderate values of τ .

Next, we present the results for the rapidity distributions $d\sigma/dQ d\eta$. As mentioned above, charmonium and bottomonium resonances complicate the calculation of Drell-Yan cross sections. Therefore usually only lepton pairs with invariant mass Q between the J/Ψ and Υ resonances and above the Υ are considered. Since the Drell-Yan event rate decreases rapidly with $\sqrt{\tau}$, it may not be possible to measure it above the Υ resonance in the medium-energy fixed-target regime accessed by the COMPASS experiment. We therefore make predictions for $\sqrt{\tau} = 0.3$ and $\sqrt{\tau} = 0.45$, corresponding to $Q = 5.7$ GeV and $Q = 8.6$ GeV, respectively. Our results are presented in Figs. 3 and 4. Again the resummed cross section and the fixed-order NLO and LO ones are shown. As before, we expand the unmatched resummed result in powers of α_s and find that the first order expansion agrees very well with the exact NLO result for $\sqrt{\tau} = 0.45$.

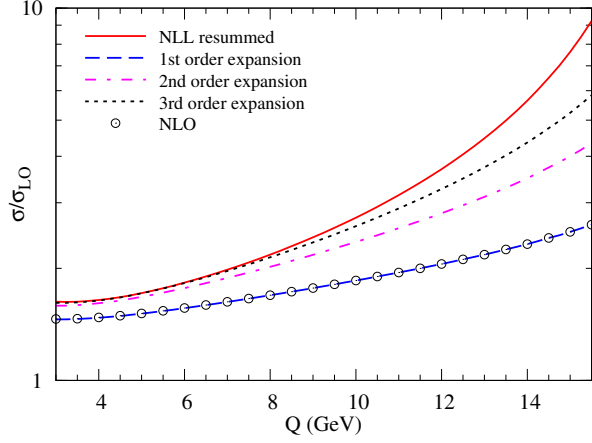


FIG. 2: “K-factors” as defined in Eq. (27) at $\sqrt{S} = 19$ GeV as functions of the lepton pair mass Q , at NLO (symbols) and for the NLL-resummed case. Also shown are the expansions of the resummed cross section to first, second and third order in the strong coupling.

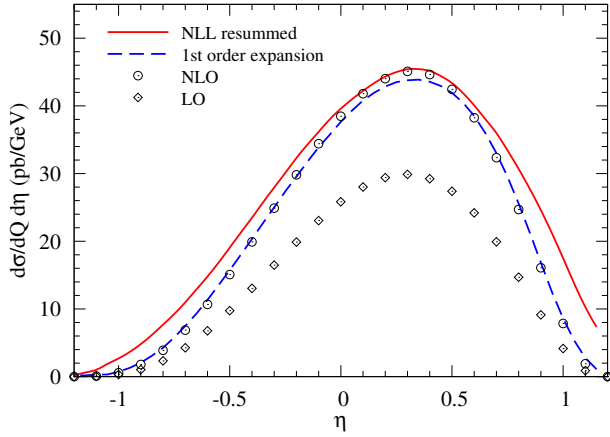


FIG. 3: Rapidity-differential Drell-Yan cross section $d\sigma/dM d\eta$ for $\pi^- p$ scattering at $\sqrt{S} = 19$ GeV and $\sqrt{\tau} = 0.3$. The LO, NLO and NLL-resummed cross sections as well as the first order expansion of the unmatched NLL-resummed cross section are shown as functions of the rapidity η of the dimuon pair.

For $\sqrt{\tau} = 0.3$, further away from threshold, the first order expansion of the threshold resummed cross section lies very slightly below the exact NLO result for central rapidities. This is due to the fact that the contributions from the threshold region $z \rightarrow 1$ do not entirely dominate the cross section in this rapidity regime. As expected from our results in Figs. 1 and 2, at fixed rapidity the threshold resummation effects become more important as τ increases, resulting in a fairly large enhancement of the resummed cross section at $\sqrt{\tau} = 0.45$. Nevertheless significant contributions from threshold resummation are still present in the cross section also for relatively modest values of τ .

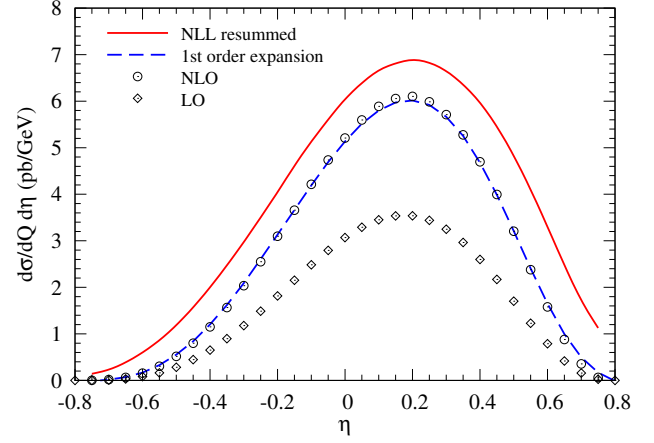


FIG. 4: Same as Fig. 3, but at $\sqrt{\tau} = 0.45$.

It is also interesting to examine to what extent resummation affects the *shape* of the rapidity dependent cross section. Figure 5 shows the ratios

$$K_{\text{res}} = \frac{\left(\frac{d\sigma_{\text{res}}}{dQ d\eta}\right)}{\left(\frac{d\sigma_{\text{LO}}}{dQ d\eta}\right)}, \quad K_{\text{NLO}} = \frac{\left(\frac{d\sigma_{\text{NLO}}}{dQ d\eta}\right)}{\left(\frac{d\sigma_{\text{LO}}}{dQ d\eta}\right)} \quad (28)$$

as functions of the pair rapidity, at $\sqrt{S} = 19$ GeV and $\sqrt{\tau} = 0.3$. One can see that K_{res} becomes very large towards the boundaries of the η interval. The resummed cross section shows a particularly sizable enhancement above the NLO one at high rapidities. This enhancement is due to the fact that at fixed τ the limit $\eta \rightarrow \eta_{\text{max}}$ corresponds to the limit $z \rightarrow 1$ at parton level. In this limit threshold logarithms become large regardless of the value of τ . Large τ and/or high rapidities in the fixed-target regime probe high momentum fractions x in the parton distribution functions. Including threshold resummation in the analysis of parton distribution functions may hence have significant effects on their extracted high- x behavior. This was examined recently in the context of the Drell-Yan process [9] and deep-inelastic lepton scattering [35].

The crucial quality test for any higher order calculation is the extent to which it reduces the scale ambiguity inherent to any perturbative QCD calculation. We examine the scale dependences of the rapidity-integrated and the rapidity-differential cross sections in Figs. 6 and 7, respectively. Again we show the LO, NLO and NLL-resummed results at $\sqrt{S} = 19$ GeV, now varying the renormalization and factorization scales between $\mu = Q/2$ and $\mu = 2Q$. Note that in Fig. 6 we have for better visibility multiplied the LO cross section by 1/2 and the resummed one by 2. Evidently for the integrated cross section the scale dependence is decreased by resummation over the whole range of invariant mass Q , whereas going from LO to NLO reduces the scale dependence only marginally. Figure 7 shows the scale dependence of the

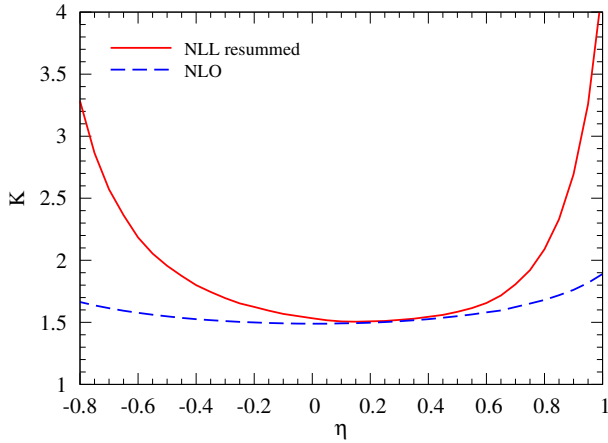


FIG. 5: Ratios K_{res} and K_{NLO} as defined in Eq. (28) at $\sqrt{S} = 19$ GeV and $\sqrt{\tau} = 0.3$, as functions of the rapidity η of the dimuon pair.

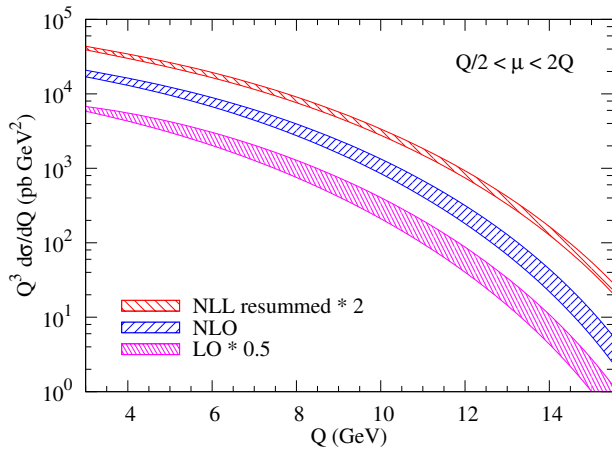


FIG. 6: Scale dependence of the LO, NLO and NLL-resummed rapidity-integrated Drell-Yan cross sections at $\sqrt{S} = 19$ GeV as function of Q . The factorization as well as the renormalization scale have been varied between $Q/2$ and $2Q$. Note that we have multiplied the LO cross section by $1/2$ and the resummed cross section by 2 .

rapidity distributions at $\sqrt{\tau} = 0.45$. Here we only show the NLL-resummed cross section and the NLO one. As one can see, the scale dependence is again significantly improved by resummation. This applies to all values of rapidity; in fact the scale dependence almost vanishes at high η after resummation.

IV. CONCLUSIONS

We have presented a phenomenological study of the Drell-Yan cross section for pion scattering off a proton target at COMPASS. In the calculation of the cross section we have resummed threshold corrections to next-to-leading logarithmic accuracy. The expansion of the

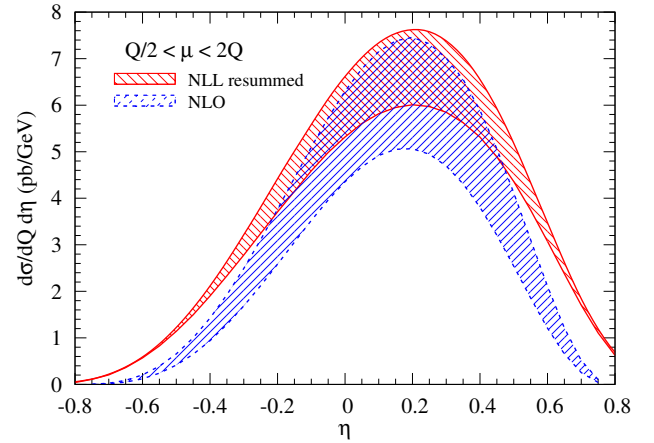


FIG. 7: Scale dependence of the NLO and NLL-resummed rapidity-differential Drell-Yan cross sections at $\sqrt{S} = 19$ GeV and $\sqrt{\tau} = 0.45$ as function of η . The factorization as well as the renormalization scale have been varied between $Q/2$ and $2Q$.

resummed cross section to $\mathcal{O}(\alpha_s)$ agrees very well with the exact fixed-order calculation. This agreement demonstrates that the large threshold logarithms indeed dominate the Drell-Yan cross section and need to be taken into account to all orders. Resumming those logarithms leads to a significant enhancement above fixed-order calculations, even for moderate values of the invariant mass Q of the lepton pair. We have also considered the rapidity dependence of the cross section. We find that even in cases where there is only a modest enhancement of the rapidity-integrated cross section by resummation, the shape of the rapidity-differential cross section is affected very strongly by resummation at sufficiently large forward or backward rapidities. Finally, we have shown that the scale dependence of the perturbative cross section is substantially reduced when threshold resummed contributions are included.

Our results overall demonstrate that threshold resummation effects will be important in the analysis of future COMPASS data. While we have only addressed the spin-averaged Drell-Yan cross section in this paper, we stress that threshold resummation effects are expected to be equally relevant also for corresponding spin-dependent cross sections, even though they may have a tendency to cancel in spin asymmetries. We also note that in the light of our study cross sections and spin asymmetries at measured transverse momentum q_\perp of the lepton pair, which will be a particular focus of the investigations at COMPASS, will require additional theoretical consideration.

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